The Robustness of Okun's Law: Evidence from Mexico. A Quarterly validation, 1985.1–2006.4[1]

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Abstract

Through the use of three Structural Time Series Models (Kalman Filter), we estimated the Okun's Law (1962) for the Mexican economy (1985.1-2006.4) and found similar results to those obtained by Loría and Ramos (2007) with annual data (1970-2004). We conclude that the Okun coefficients vary in the range 2.3-2.5 and that there is robust evidence that bidirectional causality runs between output and unemployment.

Keywords: Okun law, Structural Times Series Models, Kalman filter, causality, cointegration.

JEL Classification: C13, C22, E24.

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I. Introduction

In 1962 Arthur Okun found a statistical regularity of great relevance for the United States' economy (1947.2–1960.4), which claimed that for each percentage point of reduction in the unemployment rate, the real GDP would grow 3.3 per cent (Okun, 1962); and inversely, for each percentage point of increase in output, unemployment would vary in -0.3 points. This regularity is commonly known as 3:1, and also known as the Okun law, and since then has become a concept of great importance in modern macroeconomics. It is the consequence of relating output increase to the unemployment rate in a bi-directional way, coming out from three specifications: first differences, output gap and fitted trend and elasticity.

Okun's contribution enriches the modern macroeconomic analysis due to: a) it allows to know unemployment's variation rate in the long run, determined by structural factors such as demographics, institutions and technology; b) it provides a proxy of the natural rate of unemployment, and c) it identifies that the long run economic growth is the main factor that counteracts the reduction in employment creation capacity (see Loría and Ramos, 2007).

Okun's discovery has great importance because of its explicative capacity of economic development:

> The failure to use one year's potential fully can influence future potential GNP: to the extent that low utilization rates and accompanying low profits and personal incomes hold down investment in plant, equipment, research, housing, and education, the growth of potential GNP will retarded (Okun, 1962: 2).

From the available literature, we found that since this contribution was made, several authors have estimated different variations of this law.[2] In spite of the relevance of the

[2] Upon reviewing the literature, available works can be classified in two main categories: ***a) Theoretical-empirical studies***, in the sense that they review and discuss the estimation methods of Okun's law (in this respect see: Barreto & Howland, 1993; Altig *et al*., 1997; Attfield & Silverstone, 1998; Sogner and Stiassny, 2000; Harris & Silverstone, 2001; Crespo, 2003; Friedman y Michael, 1974; Lang y de Peretti, 2002; Prachowny, 1993; Weber, 1995; Schorderet, 2001; Knoester, 1986; Paldam, 1987), and ***b) Empirical studies***, whose main purpose is to estimate the Okun's coefficients for the case of countries, even at the level of states or regions, to know the existing interrelations between different countries by identifying reciprocities between unemployment and output. See: Abril *et al.*, 1996; Adanu, 2002; Arias *et al.*, 2002; Garavito, 2002; Lemois, 2003; Murillo y Usabiaga, 2002; Freeman, 2001; Lee, 2001; Moosa, 1997 y Schnabel, 2002.

subject and that in Mexico the problem of slow growth started since the early eighties, it is surprising that we only found three references for the Mexican economy: Chavarín (2001), González (2002) and Loría & Ramos (2007).

Our main purpose is to estimate the three Okun models (1962) for the Mexican economy using quarterly data (1985.1–2006.4), in order to prove that unemployment constraints the long run growth, and compare our results with those obtained with annual series (1970–2004) by Loría & Ramos (2007). Therefore, we corroborate that in Mexico the Okun law is validated for data of different periodicity and length. Our results indicate that there is a bi-directional causality relationship between the unemployment rate and output growth –in its three variants- and that Okun's coefficient is found in the interval of 2.3–2.5, which coincides with Loría & Ramos (2007).

In the second section we present the original Okun results. Next, we analyze the statistical properties of quarterly data that we use and point out the differences with those used by Loría & Ramos (2007). In section four we estimate the three Okun models with the methodology of structural time series models, using the Kalman filter, and contrast them with the results obtained by those authors. Additionally, we confirm the bi-directional causality in Okun's equations and test cointegration for model 3. Finally, we recover the main conclusion and outline some policy recommendations.

II. The Okun models

Okun (*op. cit.*) used three different econometric specifications to prove that there was a robust bi-directional statistical relationship between unemployment and economic growth for the economy of the United States (1947.2–1960.4) which are presented in Table 1.

Table 1
Okun models

Model	Estimation	Okun Coefficient	
		β2	1/β2
First differences (1) $\Delta U_t = \beta_1 + \beta_2 y_t + \varepsilon_t$	$\Delta U_t = 0.3 - 0.3 y_t$	0.3	3.3
Output Gap (2) $U_t = \beta_1 + \beta_2 Y_t^b + \varepsilon_t$	$U_t = 3.72 + 0.36 Y_t^b$	0.36	2.8
Fitted trend and elasticity (3) $\ln E_t = \beta_1 + \beta_2 \ln Y_t + \beta_3 t + \varepsilon_t$	$\ln E_t = 212 + 0.4 \ln Y_t - 0.32t$	0.4-0.35	2.5-2.8

Notes: U_t = Unemployment rate; y_t = Output growth; Y_t^b = Output gap = $\frac{(Y_t^P - Y_t)}{Y_t^P}$; Y_t^p = Potential output; Y_t = Actual output; E = Employment rate = $(100 - U_t)$, t = time.

Okun's empirical conclusion coming out from the first two estimations is that in the long run, the unemployment reduction has a more than proportional effect on the dynamics of GDP (1/ β_2).[3]

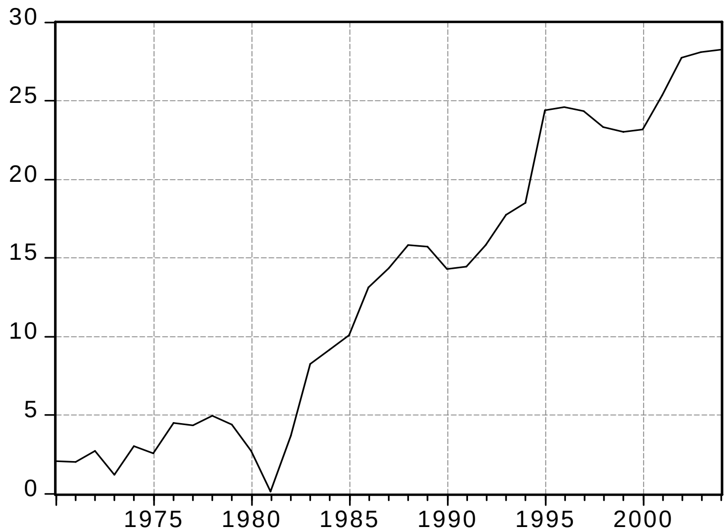
III. Mexico: output and unemployment data, 1985.1-2006.4

We use quarterly GDP (Y) data (1993 prices) and the general rate of open unemployment (U) reported by INEGI (2007a & b), both without seasonal adjustments. The latter variable differs substantially from the macroeconomic unemployment rate (TMD)[4] of Loría & Ramos (*ibid*), estimated for annual data (1970–2004). As a matter of example, towards 1970 the TMD was of 2.03 and since 1982 it began to grow until it reached 15.8% in 1988, and of 28.3% towards the year 2004 (see picture 1); while the general unemployment rate used here is stationary and currently has ranged from 2 to 5%, although it reached high figures in 1995 and 1996. See picture 2.

[3] In Loría & Ramos (*op. cit.*) the economic implications of the results depicted in this table were carefully analyzed. In addition, it is worth mentioning that (2) established the natural rate of unemployment (3.72) and (3) the output elasticity to employment.

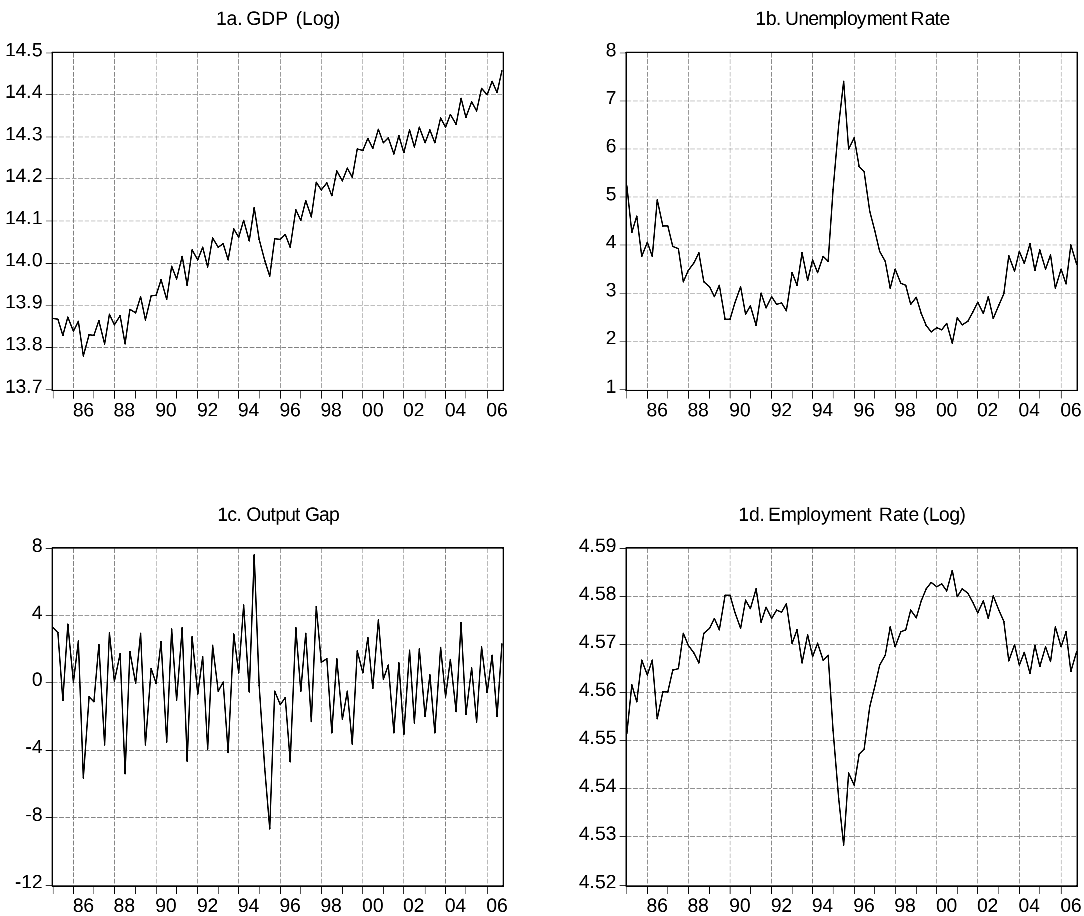
[4] $TMD = \left(\frac{PEA - PO}{PEA}\right) * 100$. where: PEA = economically active population (Conapo, 2006), PO = employees (millions of persons) in the formal sector (INEGI, 2007).

Picture 1
Mexico: macroeconomic rate of unemployment, 1970-2004



Source: Loría & Ramos (2007).

Picture 2
GDP, UNEMPLOYMENT, OUTPUT GAP AND EMPLOYMENT RATE
1985.1-2006.4



IV. Analysis and discussion

According to our main purpose, we estimated the three Okun models inversely, thus solving a serious econometric bias problem detected by Barreto & Howland (1993) in Okun's seminal article. This problem consists in estimating the current regression and afterwards solving arithmetically for the exogenous, just by doing algebra. Therefore, it does not matter regressing U on Y or the other way around. By doing this Okun claims that it is possible to find economic sense in both directions. This procedure has being followed by many authors. Accordingly, when passing directly in estimations (1) and (2) from β_2 to

$1/\beta_2$ he was able to explain –at the same statistical level- either economic growth or unemployment.

Nevertheless, in the original Okun´s models (1 and 2) there are two variables and the reading must be made as usual (from the right hand side to the left hand side), and the fact of reading inversely is not only related to the causality sense coming out from the economic theory, but also –and not less important– has to do with the properties of a joint distribution function, which refers a conditional specification of random variables of the kind

$$E(Y|X) = \alpha + \beta X,\ E(X|Y) = \gamma + \theta Y$$

Barreto and Howland (1993: 4) outline that the correct specification depends on the specific question of interest. This task determines the regression direction:

> Thus Okun's procedure [make the bi-directional reading as of β_2, our aggregate] makes sense only if the underlying structure in the model is assumed to be stable, i,e., if the parameters of the model do not change between the sample period and the date on which the GNP gap is to be predicted. If any of the structural parameters have changed in the intervening time, then the sample relationship will produce biased estimates of the GNP gap.

Thus, in order to avoid the possible problem of referred bias and since our main purpose is to prove specifically that unemployment restricts economic growth, we choose the direct estimation for the three Okun estimations. That is, we proceeded by the inverse specification to that of Okun in the following way: Y = f(U), thus the reading is direct in terms of our hypothesis.

Likewise in Loría & Ramos (2007), we estimated the three equations through the methodology of structural time series models, using the Kalman filter (Kalman, 1960). See results in table 2 and appendix C).

One advantage of this procedure is that the estimated parameter "μ_t captures the long run movements of the series involved as well as the effects that β_2 can't explain" (Loría & Ramos, 2007: 29).

Empirical evidence reports that also for this data structure and with the inverse specification of Okun, this law accomplished in Mexico. The value of the coefficients is congruent with the structure of the Mexican economy: labor intensive and low productivity.

Model (3) depicts two results with high economic meaning. On the one hand, μ_t indicates the actual rate of potential output: 2.6, similar to the figure reported by Loría, García & de Jesús (2007) of 2.5 for the span time 1980.1-2006.4. Likewise, from the parameter of the employment rate (E) we can calculate the output elasticity to employment (1/2.5892 = 0.386), that even with the methodological anticipated warnings, we can take it safely since it is congruent with the results obtained by Loría & Ramos and other applied works such as Loría (2006) and Hernández (1998) (see table 2).

Tests of unit roots (see Table 1A in the Appendix) indicate that the GPD logarithm (*ln Y*) and that of the employment rate (*ln E*) are series I(1); while the unemployment rate (*U*), GDP growth (y) and output gap (Y^b) are I(0).[5]

Table 2
Mexico: Okun estimations

Estimation		Average	
Quarterly 1985.1-2006.4	Annual, 1970-2004 (Loría & Ramos, 2007)	Quarterly	Annual
(1) $y_t = 1.1041\,\mu_t$ - $2.3538\Delta \mathrm{U}_t$	(1) $\Delta U_t = 2.349\mu_t - 0.403 y_t$	2.49	2.25
(2) $Y^B = 9.5866\,\mu_t + 2.5383\mathrm{U}_t$	(2) $U_t = 14.65\mu_t + 0.456 Y_t^B$		
(3) $\ln Y_t = 2.6115\mu_t + 2.5892 \ln \mathrm{E}_t$	(3) $\ln E_t = 0.481 \ln Y_t - 2.661\mu_t$		

All the models were estimated with GiveWin 2.3, module STAMP 6.0 (Koopman et al. 2000).

[5] This way, the problem of spurious regression could only exist in model (3) and to discard it we followed the Johansen procedure (1988). Accordingly, with a confidence level of 99% we obtained a cointegrating vector with economic sense; statistic trace 27.78 (24.6), adjustment coefficient: -0.6275 (standard error: 0.126).

Conclusion

We empirically corroborated that for quarterly data and with the use of structural time series models (using the Kalman filter), Okun’s law applies in Mexico, and the coefficient varies in the interval 2.35-2.58, which is congruent with what Loría & Ramos (2007) estimated; and furthermore, that these are magnitudes adequate for an economy that suffers from high structural unemployment and low productivity. Likewise, causality tests in the Granger sense run in a bidirectional way between unemployment and output.

In order to avoid possible biases in the estimation of the slopes of the three models, we used inverse specifications to that of Okun’s, obtaining a direct reading of the Okun coefficient and can prove in a reliable manner that unemployment constrains economic growth.

This evidence supports the results of other authors for different series and periods, which allows us to use it as a good instrument of analysis and forecasting of the economic cycle; allows us to estimate the sacrifice rate of long run unemployment; and moreover, shows that the economic policy must focus by all means in avoiding fluctuations in growth and at the same time reducing unemployment, because this way it will stimulate growth in the long run.

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Appendix

A. Basic statistics and unit roots, 1985.1-2006.4

Table 1A

	U	ln Y	Δ(ln Y)	*y*	ln E	Δ(lnE)	Y^B
Mean	3.5046	14.1067	0.0067	0.7847	4.5694	0.00019	0.0510
Median	3.3500	14.0745	-0.0015	-0.1517	4.5710	0.00055	0.0697
Std Dev.	1.0273	0.1899	0.0460	4.6480	0.0107	0.00532	2.8031
Skewness	1.3711	0.0440	0.1151	0.1706	-1.4092	-0.3741	-0.3777
Kurtosis	5.2431	1.7397	1.8257	1.8322	5.3830	3.6816	3.2045
Jarque-Bera	46.025 (0.000)	5.851 (0.053)	5.191 (0.075)	5.365 (0.068)	49.953 (0.000)	3.714 (0.156)	2.246 (0.325)
ADF	-2.692	3.913[1]	-4.036[1*]	-3.153[2]	0.100[3]	-4.287	-7.048[4]
DF-GLS	-1.776[5]	-2.618	-3.640[2*]	-3.230[6]	-2.513	-2.258	-6.024
PP	-3.785	3.691[8]	-34.907[3*]	-4.728[9]	3.802	-11.704	-16.325[10]
KPSS	1.235	202.862	0.155	0.432[11]	1.214	0.044	0.173

Tests are non-significant at levels. ADF with four lags and intercept are valid at 90%; DF-GLS with four lags, trend and intercept; PP with four lags and intercept, valid at 99%. KPSS with four lags and intercept.
[1] Trend and intercept; [2] with 5 lags valid at 95%; [3] with 5 lags, without trend or intercept; [4] with 5 lags; with no trend or intercept, valid at 99%; [5] with intercept; [6] with 5 lags, valid at 95%; [7] with 5 lags at 99%; [8] with 5 lags, without trend or intercept; [9] with 5 lags; [10] No intercept nor trend; [11] with 5 lags.
Tests in differences are valid at 99% of confidence. ADF with four lags and no trend nor intercept; DF-GLS is non-significant with three lags, trend and intercept; PP with four lags, no intercept nor trend; KPSS with four lags and intercept.
[1*]With intercept only; [2*] with 4 lags and intercept; [3*] with intercept; three lags.

It is concluded that U, Δ(ln Y), $Y^B \sim I(0)$, and that ln Y, ln E ~ I(1).

B. Causality Test

Table 1B
Granger Causality Test, 1985.1-2006.4 for an
Unrestricted VAR(5)

VAR Model	Ho: Not causality	χ^2 (5)	Probability
[1]	ΔU_t does not cause $\dot{y}_t$	21.39	0.0007
	$\dot{y}_t$ does not cause ΔU_t	28.42	0.0000
		χ^2 (5)	
[2]	U_t does not cause Y_t^B	20.92	0.0008
	Y_t^B does not cause ΔU_t	43.88	0.0000
		χ^2 (5)	
[3]	$\ln E_t$ does not cause $\ln Y_t$	27.79	0.0000
	$\ln Y_t$ does not cause $\ln E_t$	17.47	0.0037

Models 1 and 3 include a GDP shock dummy and an inflation dummy for 1986.3 and 1995.2.

C. Statistical tests of incorrect specification

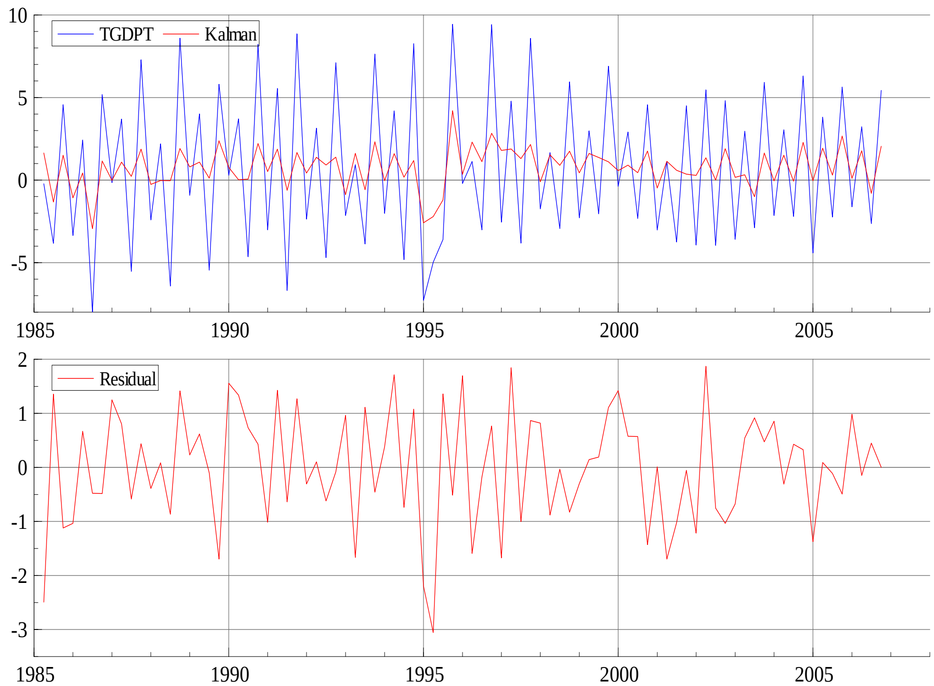
Model 1. First differences

$$\Delta U_t = \begin{pmatrix} 1 & 0 & y_t \end{pmatrix} x_t + + \begin{pmatrix} \sigma_\varepsilon^2 & 0 & 0 \end{pmatrix} \begin{pmatrix} \varepsilon_t \\ \zeta_t \\ \xi_t \end{pmatrix}$$

$$x_t = \begin{pmatrix} \mu_t \\ \beta_t \\ \delta_t \end{pmatrix} = \begin{pmatrix} 1 & 1 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} x_{t-1} + \begin{pmatrix} 0 & \sigma_\zeta^2 & 0 \\ 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} \varepsilon_t \\ \zeta_t \\ \xi_t \end{pmatrix}$$

Table 1C
Mexico: Okun's Law, 1985.1-2006.4

		$y_t = 1.1041\,\mu_t - 2.354\Delta\mathrm{U}_\mathrm{t}$
R^2		0.882962
N		3.2632 (0.1956)
DW		2.3062
r	(1)	-0.21520 (0.8296)
	(8)	0.066376 (0.9471)
Q	(8,6)	21.292 (0.0016)
H	(34)	0.74192 (0.8058)


TGDPT
Kalman
Residual
10
5
0
-5
2
1
0
-1
-2
-3
1985
1990
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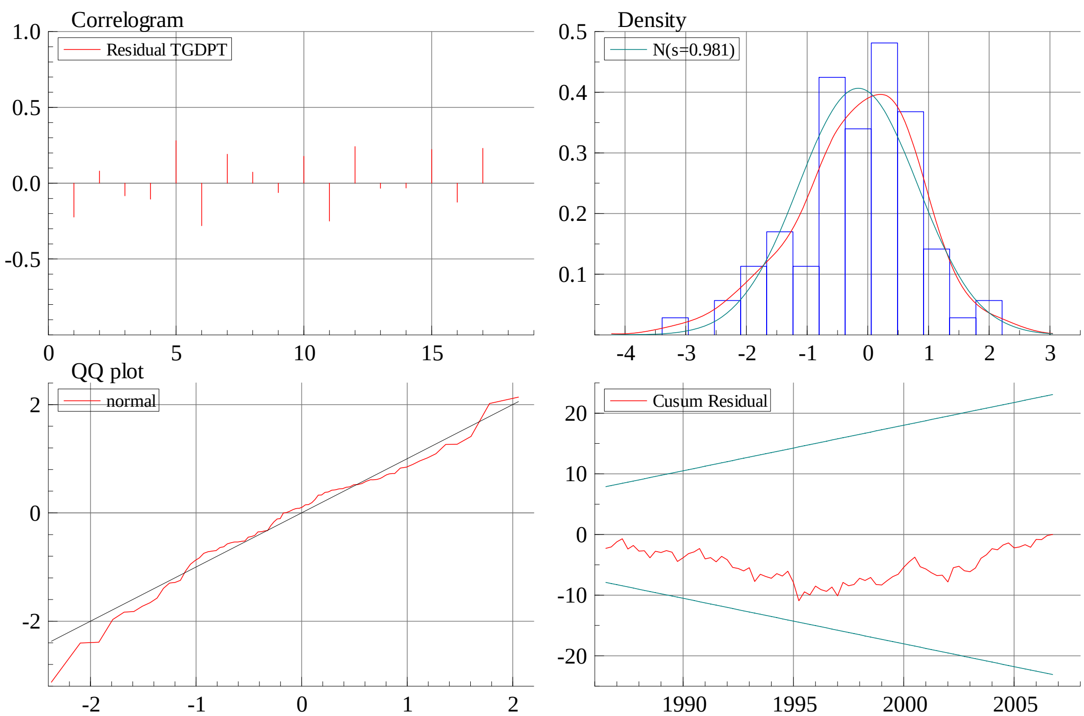
2000
2005


Correlogram
Residual TGDPT
Density
N(s=0.981)
QQ plot
normal
Cusum Residual
1.0
0.5
0.0
-0.5
0
5
10
15
0.5
0.4
0.3
0.2
0.1
-4
-3
-2
-1
0
1
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20
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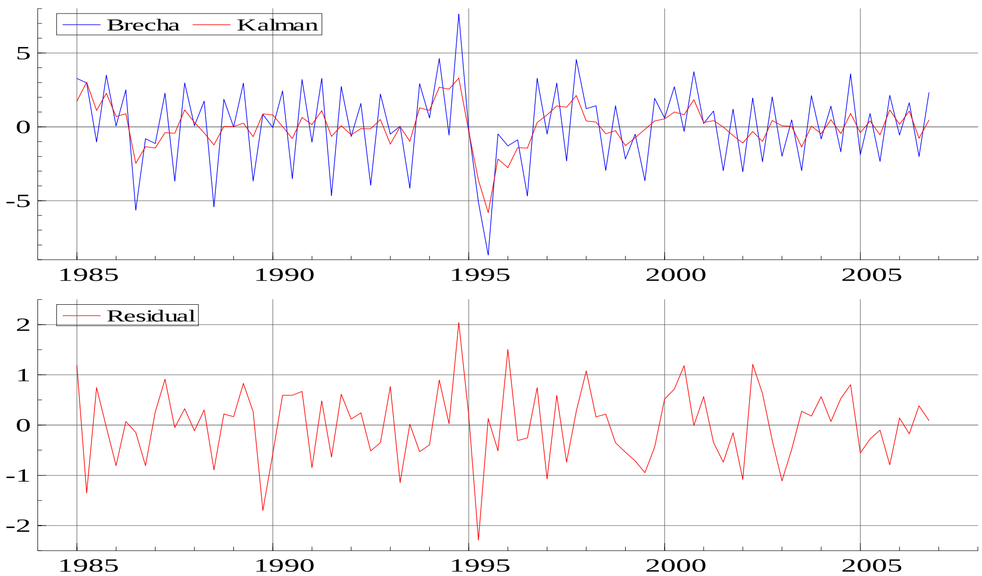
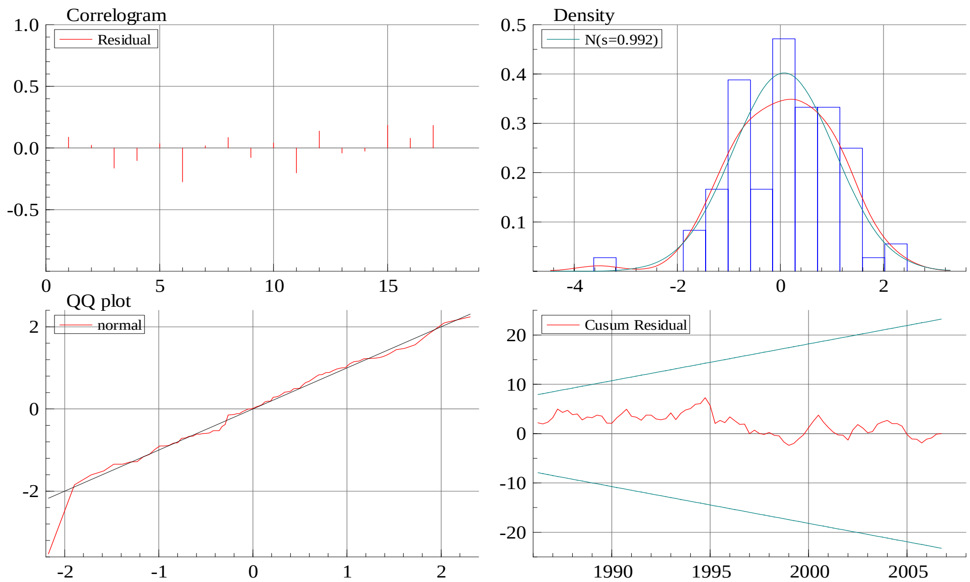
Model 2. Output Gap

$$U_t = \begin{pmatrix}1 & 0 & Y_t^B\end{pmatrix} x_t + + \begin{pmatrix}\sigma_\varepsilon^2 & 0 & 0\end{pmatrix}\begin{pmatrix}\varepsilon_t \\ \zeta_t \\ \xi_t\end{pmatrix}$$

$$x_t = \begin{pmatrix}\mu_t \\ \beta_t \\ \delta_t\end{pmatrix} = \begin{pmatrix}1 & 1 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1\end{pmatrix} x_{t-1} + \begin{pmatrix}0 & \sigma_\zeta^2 & 0 \\ 0 & 0 & \sigma_\xi^2\end{pmatrix}\begin{pmatrix}\varepsilon_t \\ \zeta_t \\ \xi_t\end{pmatrix}$$

		$Y^B = 9.5866\,\mu_t + 2.5383\mathrm{U}_t$
R^2		0.705865
N		4.2233 (0.1210)
DW		1.7490
r	(1)	0.086169 (0.9313)
	(8)	0.076475 (0.9390)
Q	(8,6)	10.342 (0.1110)
H	(27)	1.0967 (0.4061)



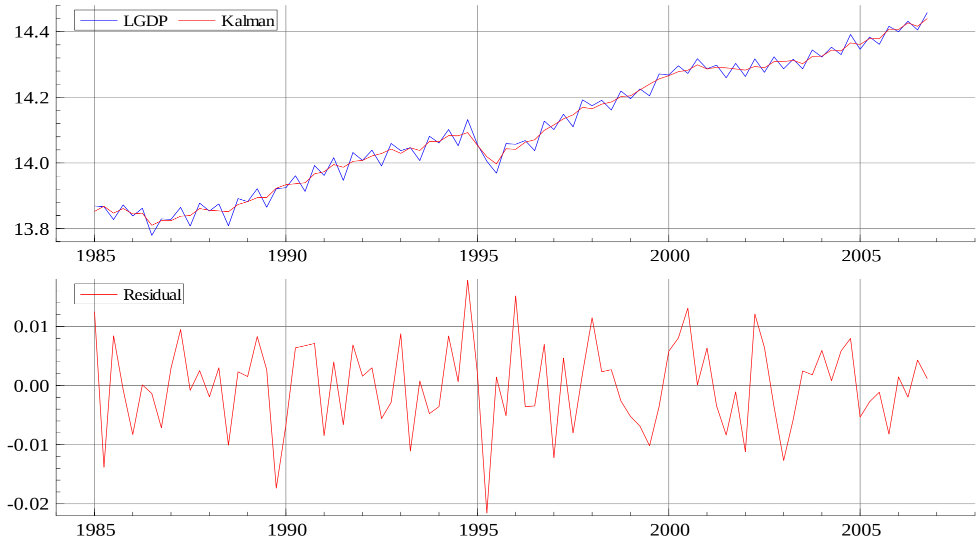

Correlogram
Residual
1.0
0.5
0.0
-0.5
0
5
10
15
Density
N(s=0.992)
0.5
0.4
0.3
0.2
0.1
-4
-2
0
2
QQ plot
normal
2
0
-2
-2
-1
0
1
2
Cusum Residual
20
10
0
-10
-20
1990
1995
2000
2005

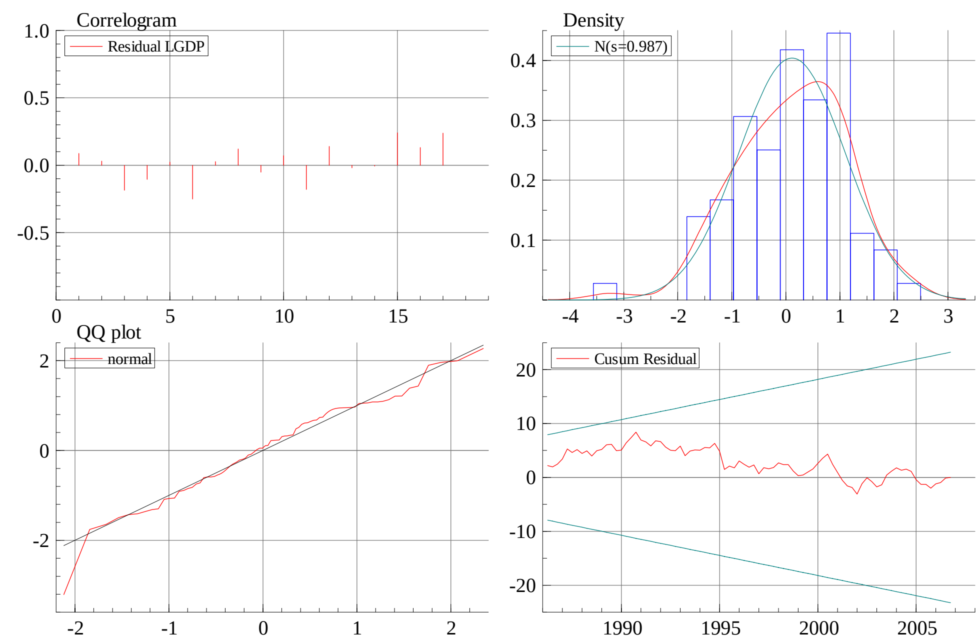
Model 3. Fitted Trend and Elasticity

$$\ln E_t = \begin{pmatrix} 1 & 0 & \ln Y_t \end{pmatrix} x_t + + \begin{pmatrix} \sigma_\varepsilon^2 & 0 & 0 \end{pmatrix} \begin{pmatrix} \varepsilon_t \\ \zeta_t \\ \xi_t \end{pmatrix}$$

$$x_t = \begin{pmatrix} \mu_t \\ \beta_t \\ \delta_t \end{pmatrix} = \begin{pmatrix} 1 & 1 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} x_{t-1} + \begin{pmatrix} 0 & \sigma_\zeta^2 & 0 \\ 0 & 0 & \sigma_\xi^2 \end{pmatrix} \begin{pmatrix} \varepsilon_t \\ \zeta_t \\ \xi_t \end{pmatrix}$$

		$\ln Y_t = 2.6115\mu_t + 2.5892\ln E_t$
R^2		0.993693
N		2.7109 (0.2578)
DW		1.7490
r	(1)	0.085145 (0.9321)
	(8)	0.10788 (0.9141)
Q	(8,6)	10.480 (0.1058)
H	(34)	1.1143 (0.3771)


LGDP
Kalman
14.4
14.2
14.0
13.8
1985
1990
1995
2000
2005
Residual
0.01
0.00
-0.01
-0.02
1985
1990
1995
2000
2005


Correlogram
Residual LGDP
1.0
0.5
0.0
-0.5
0
5
10
15
Density
N(s=0.987)
0.4
0.3
0.2
0.1
-4
-3
-2
-1
0
1
2
3
QQ plot
normal
2
0
-2
-2
-1
0
1
2
Cusum Residual
20
10
0
-10
-20
1990
1995
2000
2005